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Three units of instruction were given to 20 four-year-old children and 34 five-year-old children. Unit I was designed to develop the children's ability to establish a length relation between curved lines; Unit II, to develop ability to conserve length; and Unit III, to develop ability to conserve length relations. Testing of the children occurred between Units I and II, and after Unit III. Three tests were administered during the testing session: (1) a six-item test designed to measure the children's ability to conserve length; (2) an 18-item test to measure the children's ability to conserve length relations; and (3) a six-item test to measure the children's ability to deal with transitivity of length. The test results indicated that (1) the ability to conserve length as measured in this study is not a necessary or sufficient condition for the ability to use transitivity of length; (2) ability to conserve length relations may be necessary for transitivity; and (3) ability to conserve length is not a necessary or sufficient condition for conservation of length relations. (WD)

A Study of The Interrelationships of Conservation of Length Relations, Conservations of Length, and Transitivity of Length Relations of the Age of Four and Five Years.

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INTRODUCTION

When writing about Piaget's conservation problems, Elkind has stated that "...every conservation problem assesses two different forms of conservation..." (1, p. 15). In order to clarify this statement, he presented the following symbolic description of the problems.

> Regardless of the content of these problems, they routinely involved presenting the subject with a variable (V) and a standard (S) stimulus that are initially equivalent in both the perceptual and the quantitative sense. The subject is then asked to make a judgement regarding their quantitative equivalence. Once the judgement is made, the variable stimulus is subjected to a transformation, $V \rightarrow V'$, which alters the perceptual but not the quantitative equivalence between variable and standard. After completion of the transformation, the subject is asked to judge the quantitative equivalence between the standard and the transformed variable (1, p. 16).

Involved in the problem is a judgement of conservation of identity, which Elkind describes as follows:

> suppose in the weight conservation problem...one employed only a single ball of clay which was then rolled into a sausage, and the child was asked whether the clay was now the same weight as before.... This would be a direct assessment of what will hereafter be called the "conservation of identity." (1, p. 16)

Elkind goes on to say,

> It is probably true...that from the point of view of the subject, the conservation of identity is a necessary condition for the conservation of equivalence (1, p. 17).

Elkind also believes that conservation of equivalence demands a process of reasoning not present in conservation of identity problems. He states, "It is clear...that the child must employ a form of deduction from immediate past experience to arrive at the conservation of equivalence." (1, p. 22).

Smedslund, in a study of concrete reasoning, reported that in tests which involved the relation "longer than", all subjects who passed a transitivity test also passed a conservation test, with the exception of one child. Smedslund's test items of conservation involved the Müller Lyer (2). Elkind argues that in items involving the Müller Lyer, it is impossible to assess conservation of identity (1, p. 22). This leads one to the tentative conclusion that Smedslund was studying conservation of an order relation. It may therefore be conjectured that conservation of identity, as Elkind views it, is a necessary condition of transitivity of a relation.

Regardless of whether Elkind is speaking of conservation of identity or conservation of equivalence, he sees the child as being asked to make quantitative judgments. If, in the case of the relations "the same length as", "longer than" and "shorter than" for curves of finite length, one assumes that children are asked to make quantitative judgements in conservation problems, than the relation should be defined as follows:

> If S and V are curves of finite length, then S is the same length as V if, and only if, $L(S)=L(V)$, where $L(S)$ and $L(V)$ are numbers which denote the lengths of S and V respectively. S is shorter than V if, and only if, $L(S)<L(V)$. S is longer than V if, and only if, $L(S)>L(V)$.

If T is a transformation which is length preserving, then $L(V)=L[T(V)]$. In the case that S is the same length as V, $L(S)=L(V)$ and hence $L(S)=L[T(V)]$.

If the children cannot associate a number with S and V, then there seems to be little reason to expect them to be able to make a judgment regarding the quantitative equivalence of S and V or of V and T(V). Therefore, under this condition, there is no reason to expect children to conserve a quantitative equivalence between S and V; i. e., deduce that S and T(V) are of the same length; nor "conserve identity" between V and T(V).

Moreover, the initial relation between S and V need not be an equivalence relation. It may be, in fact, an order relation (e.g. L(S)<L(V) may hold, where L(S) and L(V) are numbers associated with the curves S and V, respectively.)

In this study, it was not assumed that children at the ages of four and five years are able to make quantitative judgments. Hence, the relations "the same length as", "longer than" and "shorter than" were defined as follows:

> A is the same length as B if, and only if, when the curves (or their transforms) lie on a line in such a way that two endpoints coincide (left or right), the two remaining endpoints coincide. A is longer than B if, and only if, the remaining endpoint of B coincides with a point between the endpoints of A. Also, in this case, B is shorter than A.

It is essential to note that the definitions are given entirely in terms of a line, the endpoints of curves, betweenness for points and coincident points on a line. In the case of the relation "the same length as", the reflexive, symmetric and transitive properties hold. For the relation "longer than" or "shorter than", the non-reflexive, asymmetric and transitive properties hold. The following statements are all logical consequences of the manner in which the relations are defined:

(a) A shorter (longer) than B is equivalent to B longer (shorter) than A.

(b) A the same length as B implies A is not shorter (longer) than B.

(c) A shorter (longer) than B implies A not longer (shorter) than B.

In the context of the latter definitions of the three relations "longer than", "shorter than" and "the same length as", it is possible to view conservation of identity as a test of the reflexive property of "the same length as". It is not sufficient, however, since if a child does judge A and T(A) (T a length preserving transformation) to be of the same length, but also judges T(A) to be longer than A, a contradiction is present. It is therefore necessary to view conservation of identity in the context of the non-reflexive property of "longer than" and "shorter than".

Extrapolating from Elkinds view that conservation of equivalence involves a form of deduction from immediate past experiences not totally explainable by conservation of identity, it is assumed that conservation of the <u>relations</u> "longer than", "shorter than" and "the same length as" all involve the same <u>form</u> of deduction. If "~" denotes "the same length as"; S and V denote curves of finite length; and T denotes a length preserving transformation, then from a childs establishing S~V, it is necessary he/she deduces that S~T(V) in order to conserve the relation. It is not sufficient since, for example, if the child also concluded that $S \prec T(V)$ (where "$\prec$" denotes "shorter than"), a contradiction would be present. It is therefore necessary to view conservation of the relation in the context of the asymmetric property and consequences of the relations. Similar statements may also be made in the case of transitivity. From establishing A~B and B~C, where A, B and C denotes open curves of finite length and "~" denotes "the same length as", if a child deduces that A~C, he must also deduce that $A \not\prec C$ or $A \not\succ C$, where "$\prec$" and "$\succ$" denote "shorter than" and "longer than", respectively.

The way in which conservation of identity and conservation of length relations are viewed in this study precludes the possibility that, on a logical basis, conservation of identity is a necessary or sufficient condition for conservation of length relations or for transitivity. It is in fact true that in the case of "the same length as", the reflexive property can be deduced from the symmetric and transitive properties. The non-reflexivity of "longer than" and "shorter than" is obtainable as a logical consequence of the asymmetric property and does not imply transitivity.

If a child establishes a relation ' ' between two curves in accordance to the definition given earlier, then the conservation of "" may be a realization by the child that the relation, and no other relation, obtains regardless of the proximity of the curves. Viewed in the manner, the conservation of "~" is essential for the transitive property of ' ".

The definition of the pupil abilities considered in the study are given below. A word of clarification is in order. It must be pointed out that the phrase "conservation of length" refers to a test of the reflexive and non-reflexive properties. "Length" is substituted for "identity" not to introduce confusion, but for the purpose of denotation.

Definitions of Pupil Abilities

In the following definitions, A, B, and C represent open curves of finite length. A curve and a physical representation of it's trace will not be distinguished. The possibility of "straightening" a curve will be assumed.

(1) Length comparison between two curves:

Given two curves A and B, a child is said to be able to establish a length relation "~" ("longer than", "shorter than", or "the same length as") between A and B if and only if
- (a) The child places each curve on a line in such a way that two endpoints (left or right) coincide.
- (b) Compares the relative position of the two remaining endpoints, and then,
- (c) On a basis of (a) and (b), deduces that A·B, if in fact it is true that A~B.

(2) Conservation of length of a curve (reflexive and non-reflexive properties):

Given a curve A and a length preserving transformation T, a child is said to be able to conserve the length of A if, and only if, he deduces that A and T(A) are of the same length and A and T(A) are not of different lengths.

(3) Conservation of a length relation between two curves:

A length relation between two curves A and B is conserved by a child if, and only if, the relation is (a) established by the child and then (b) retained, regardless of any length-preserving transformation on one or both of the curves.

(4) Transitive property of length relations:

A child is said to be able to use the transitive property of the relations "~" (where "~" may be replaced by "longer than", "shorter than", or the same length as") if, and only if, from establishing that A~B and B~C, he is able to deduce that A~C, and that no other relation holds between A and C.

(5) Symmetric (asymmetric) property of length relations:

A child is said to be able to use the symmetric property of "~" (where "~" may be replaced by "the same length as") if, and only if, from establishing that A~B, he is able to deduce that B~A. A child is said to be able to use the asymmetric property of "α" (where "α" may be replaced by "shorter than") if, and only if, from establishing AαB, he is able to deduce that B|A (likewise for "longer than").

(6) Consequences:

(A) A child is said to be able to use consequence (a) if, and only if, from establishing that A is shorter than B, he is able to deduce that B is longer than A, or vice versa.

(B) A child is said to be able to use consequence (b) if, and only if, from establishing A the same length as B, he is able to deduce that A is not longer (shorter) than B.

(C) A child is said to be able to use consequence (c) if, and only if, from establishing A shorter (longer) than B, he is able to deduce that A is not longer (shorter) than B.

The relations and their properties should not be presented to the pre-school child by the use of words alone. The relations need to be operationally defined for the child, i.e., defined by physical operations with concrete objects. Once the initial definition is made, the physical operation may be performed by the child.

The following questions are of basic concern for this study:

(1). Is the ability to conserve length necessary (sufficient) for children to be able to use the transitive property with or without having had formal experience in conserving length?

(2). Is the ability to conserve length relations necessary (sufficient) for children to be able to use the transitive property with or without having had formal experiences in conserving length relations?

(3). Is the ability to conserve length relations necessary (sufficient) for children to be able to conserve length with or without having had formal experiences in each?

PROCEDURE

The procedure will be given in highly abbreviated form. A short description of the tests, instructional units and subjects are included.

The Test

Conservation of Length

A six item test was constructed to measure the ability of children to conserve length. Three of the items involved the reflexive property of the "same length as" and three of the items involved the non-reflexive property of "longer than" or "shorter than". Five different material sets were employed. A child had to obtain a five or a six on the test in order to meet performance criterion. The responses obtained from the children were "yes"-"no" responses. In order to score the item involving the reflexive property correctly, a child had to respond "yes". In order to score those items involving the non-reflexive property correctly, a child had to respond "no". The items were randomized independently for each child. The children were tested individually on a one-to-one basis.

Conservation of Length Relations

An eighteen item test was constructed. Nine of the items were constructed to measure the ability of children to conserve length relations without involving any properties or consequences of the relations. These items were constructed in such a way that a child had to first establish a relation between two curves and then, in the face of a perceptual conflict, conserve that relation. The question was worded in such a way that the terms of the relation were not interchanged and the relation was not changed (e.g., if the child established the relation A is the same length as B, then either A or B was transformed and the question asked was "Is A still the same length as B?") Each of these nine questions required a "yes" response. Three items were constructed to test conservation of "the same length as", three conservation of "longer than" and three conservation of "shorter than".

The remaining nine items involved the asymmetric property or consequences of the relations. Four of the nine items involved the asymmetric property and five of the items involved a change of the relation. The item format was the same as that outlined immediately above, but with a different question. The same material sets were used in each set of nine items.

The eighteen items were randomly administered to each child on a one-to-one basis. The randomization was conducted independently for each child. In order to meet performance criterion, a child must score six "yes" items correctly and six "no" items correctly.

Transitivity

A six item test was constructed. Three of the items involved transitivity of "shorter than", "longer than" and "the same length as" without a change of relation or terms of the relation in the question (e.g., if the child established

A~B and B~C, the experimenter asked, "Is A~C?"). The remaining three items involved a change of relation (e.g., if the child established A~B and B~C, the experimenter asked, "Is A~C?). The correct response to each of the first three items was "yes" and the correct response to each of the second three items was "no". It must be pointed out that in the second three items, the ability of the children to use a consequence of the relations under consideration was involved.

The six items were randomly administered to each child on a one-to-one basis. The randomization was conducted independently for each child. In order to meet criterion performance, a child must make at least five correct responses <u>after</u> establishing the correct relation twice for each item.

The Instructional Units

Three instructional units were designed. Unit I was designed to develop the ability of children to establish a length relation between curves; Unit II was designed to develop the ability of children to conserve length; Unit III was designed to develop the ability of children to conserve length relations. Unit II preceded Unit III due to Elkind's claim that conservation of identity is necessary for conservation of equivalence.

Small group instructional procedures were utilized where an instructional group generally consisted of six children. For any one child, Unit I consisted of seven 20-30 minute sessions; Unit II consisted of three 20-30 minute sessions; Unit III consisted of five 20-30 minute sessions. Two administrations of the three testing instruments described above were performed; the first between Unit I and II and the second after Unit III.

Subjects

The subjects for the study were 20 four year old children and 34 five year old children in the Suder Elementary School, Jonesboro, Georgia.

RESULTS

The statements made in this section are relative to only those children in the sample. The results are organized by basic questions.

(1) Is the ability to conserve length necessary (sufficient) for chilren to be able to use the transitive property with or without having had formal experience in conserving length?

Prior to formal experiences in conserving length, two students met criterion on the Conservation of Length Test but neither met criterion on the Transitivity Test. Of the five children who met criterion on the Transitivity Test, none met criterion on the Conservation of Length Test. After formal experiences, only one student out of the fourteen who met criterion on the Conservation of Length Test met criterion on the Transitivity Test. This student did not meet the criterion for conservation of length relations. Since ten five year old children met criterion on the Transitive Test, it is quite apparent that the ability to conserve length as measured here is not a necessary nor a sufficient condition for the ability to use transitivity of length relations. This observation is quite consistent with the fact that the reflexive property of "the same length as" does not imply the transitive property of "the same length as" nor does the non-reflexive property of "longer than" or "shorter than" imply the transitive property of these two relations, on a logical basis. Conversely, the transitive property of "longer than" or "shorter than" does not imply the non-reflexive property of these two relations.

(2) Is the ability to conserve length relations necessary (sufficient) for children to be able to use the transitive property with or without having had formal experiences in conserving length relations?

Before formal experiences in conservation of length relations, only one out of the six students who met criterion on the Conservation of Length Relations Test met criterion on the Transitivity Test. Of five students who met criterion for the Transitivity Test, only one met criterion for the Conservation of Length Relations Test. After formal experiences in conservation of length relations, seven of the nineteen students meeting criterion on the Conservation of Length Relations Test met criterion on the Transitivity Test. Since only ten children met criterion on the Transitivity Test, it seems that after formal experiences, conservation of length relations involving properties and consequences may be necessary for transitivity. The fact that two of three children who meet criterion on the Transitivity Test but not on the Conservation of Length Relations Test, did not meet criterion on the Conservation of Length Test indicates an inaccurate assessment of transitivity. The above data are consistent with Smedslund's observation that what he calls conservation of length is a necessary condition for what he calls transitivity.

(3) Is the ability to conserve length necessary (sufficient) for children to be able to conserve length relations with or without having had formal experiences in each?

Prior to formal experiences, the two children who met criterion on the Conservation of Length Test did not meet criterion on the Conservation of Length Relations Test and the six children who met criterion on the latter test did not meet criterion on the former test. After formal experiences, seven of the nineteen children who met criterion on the Conservation of Length Relations Test met criterion on the Conservation of Length Test, and seven of the fourteen children who met criterion on the Conservation of Length Test met criterion on the Conservation of Length Relations Test.

It appears that conservation of length involving both the reflexive and non-reflexive properties as measured in this study is not a necessary nor a sufficient condition for conservation of length relations involving properties and consequences. This observation is consistent with the logical interrelationships of the properties of the relations. However, the data does not contradict the fact that conservation of length involving only the reflexive property may precede conservation of length relations.

BIBLIOGRAPHY

1 Elkind, David. Piaget's Conservation Problems. Child Development, 1967, 38, 15-27.

2 Smedslund, Jan. Concrete Reasoning: A Study of Intellectual Development, Monographs of the Society for Research in Child Development. Serial No. 93, 1964.